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# MOTION OF AN AIRFOIL NEAR A FLAT SCREEN

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The problem of motion of an airfoil near a screen is not only of practical but also of theoretical interest. The proximity of the screen changes the nature of the dependence of the aerodynamic characteristics on the angle of attack and airfoil shape which prevail in an unlimited stream. Moreover, a strong mutual influence of the parameters of the problem is observed, casting doubt on its solution by methods of thin-wing linear theory [1, 2].

In the general formulation, without any simplifications, the problem was solved in [1, 3] by the method of conformal mappings. However, the results of calculations by this method were published only for the case of a plate [2, 3].

In the present paper, the boundary-value problem of flow past an airfoil moving near a screen is reduced to a system of integral equations that do not degenerate in the limiting case of an infinitely thin airfoil. These equations are solved by an improved method of discrete vortices, which permits a highly accurate calculation of distributed and overall aerodynamic characteristics for airfoils of any thickness. The calculation results presented show a significant influence of airfoil thickness on the nature of the dependence of the aerodynamic characteristics on the angle of attack and on the distance of the airfoil from the screen.

**1.** Consider an airfoil L moving at a constant velocity above a flat screen in an ideal fluid. We introduce the coordinate system Oxy fixed in the airfoil, the O axis being directed along the screen. Let $V_\infty$ be the velocity of inverted motion of the fluid at an infinite distance from the airfoil, b, the airfoil chord, H, the distance of its trailing edge from the screen, and $\alpha$, the angle of attack (Fig. 1). The corresponding boundary-value problem for complex velocity $\bar{V}(z)$ in the plane of complex variable z can be reduced to some integral equations in the tangential component of velocity $V_S(z)$ on contour L. Such equations may, in particular, be the following:

$$\mathrm{Im}\{e^{i\theta(z)}\overline{V}_0(z)\} = 0,\ z \in L; \tag{1.1}$$

$$\frac{1}{2} V_s(z) = \mathrm{Re}\{e^{i\theta(z)}\overline{V}_0(z)\},\ z \in L. \tag{1.2}$$

Here $\theta(z)$ is the angle between the tangent to the airfoil at point z and the Ox axis;

$$\overline{V}_0(z) = V_\infty + \frac{1}{2\pi i}\int_L K(z, \zeta)V_s(\zeta)e^{-i\theta(\zeta)}d\zeta; \tag{1.3}$$

$$K(z, \zeta) = \frac{1}{z-\zeta} - \frac{1}{z-\bar{\zeta}}. \tag{1.4}$$

The particular integral in (1.3) is understood in the sense of the Cauchy principal value.

Note that Eqs. (1.1) and (1.2) can be solved independently of one another. In the limiting case of flow around a small airfoil profile, these equations degenerate, taking the same form on the upper and lower sides of the airfoil.

As in [4], from independent Eqs. (1.1) and (1.2) one can obtain a system of two simultaneous integral equations that do not have a parametric singularity associated with the airfoil thickness. This system is of the form

$$\mathrm{Im}\{e^{i\theta(z_1)}\overline{V}_0(z_1) - e^{i\theta(z_2)}\overline{V}_0(z_2)\} = 0; \tag{1.5}$$

$$\frac{1}{2}\{V_s(z_1) - V_s(z_2)\} = \mathrm{Re}\{e^{i\theta(z_1)}\overline{V}_0(z_1) - e^{i\theta(z_2)}\overline{V}_0(z_2)\}, \tag{1.6}$$

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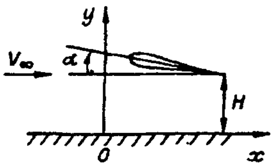


Fig. 1

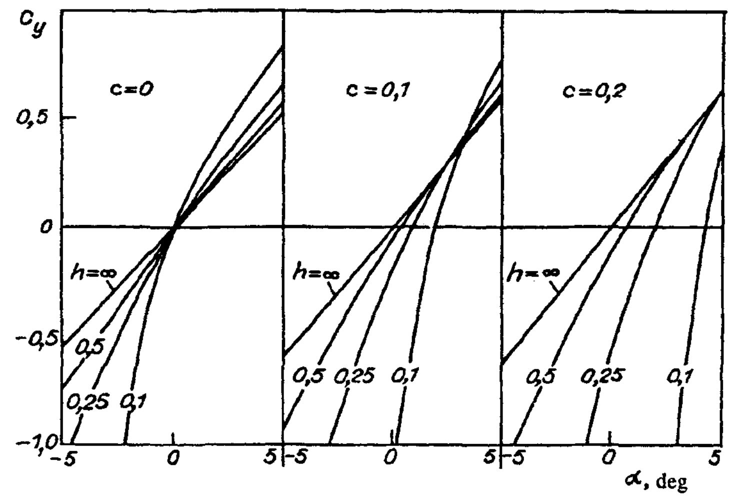


Fig. 2

where $z_1 \in L_1$; $z_2 \in L_2$; contours $L_1$, $L_2$ determine the upper and lower sides of the airfoil, and points $z_1$, $z_2$ correspond to one another in the sense that in the limiting case of an infinitely thin airfoil (airfoil profile), they become a single point.

The system of Eqs. (1.5), (1.6) will be solved by the discrete vortex method. For this purpose, we introduce the intensities of vortex layers $\gamma_1(z_1)$, $\gamma_2(z_2)$ on contours $L_1$, $L_2$, assuming that $\gamma_r(z_r) = -V_s(z_r)$, $r = 1, 2$. The vortex layers on $L_1$, $L_2$ are divided into N elements. We replace the vortex layer on each element by a single discrete vortex $\Gamma_m^{(r)}$, placing it at the point $z_m^{(r)}$ ($m = 1, \ldots, N$; $r = 1, 2$). We select on the same elements certain other points $z_{0k}^{(r)} \in L_r$ ($k = 1, \ldots, N$; $r = 1, 2$) and require that Eqs. (1.5), (1.6) hold at these points. Then Eqs. (1.5), (1.6) with $z_r = z_{0k}^{(r)}$ ($k = 1, \ldots, N$; $r = 1, 2$) become a system of 2N equations in which, in accordance with (1.3), (1.4),

$$\overline{V}_0(z_{0k}^{(r)}) = V_\infty - \frac{1}{2\pi i}\int_{L_1} K(z_{0k}^{(r)}, \zeta_1)\gamma_1(\zeta_1)e^{-i\theta(\zeta_1)}d\zeta_1 - \frac{1}{2\pi i}\int_{L_2} K(z_{0k}^{(r)}, \zeta_2)\gamma_2(\zeta_2)e^{-i\theta(\zeta_2)}d\zeta_2. \tag{1.7}$$

The integral expressions in (1.7) will be approximated by quadrature formulas containing discrete vortices. Let us assume that the points $z_{0k}^{(r)}$, $z_m^{(r)}$ belong to the same contour. Then the integrals under consideration may be approximated by the formula [5]

$$\frac{1}{2\pi i}\int_{L_r} K(z_{0k}^{(r)}, \zeta_r)\gamma_r(\zeta_r)e^{-i\theta(\zeta_r)}d\zeta_r = \frac{1}{2\pi i}\sum_{m=1}^{N}\Gamma_m^{(r)}K(z_{0k}^{(r)}, z_m^{(r)}). \tag{1.8}$$

Let $z_{0k}^{(r)}$, $z_m^{(p)}$ lie on different contours ($p \neq r$; $p, r = 1, 2$). In that case, an approximation of the type of (1.8) proves insufficient if point $z_m^{(p)}$ is near contour $L_r$. Therefore, for $p \neq r$, the following quadrature formula was chosen:

$$\begin{gathered}\frac{1}{2\pi i}\int_{L_r} K(z_{0k}^{(p)}, \zeta_r)\gamma_r(\zeta_r)e^{-i\theta(\zeta_r)}d\zeta_r = \frac{1}{2\pi i}\sum_{m=1}^{N}\Gamma_m^{(r)}K(z_{0k}^{(p)}, z_m^{(r)}) + \\ + \frac{1}{2\pi}\Gamma_k^{(r)}g_k^{(r)}(z_{0k}^{(p)}),\ p \neq r,\end{gathered} \tag{1.9}$$

where

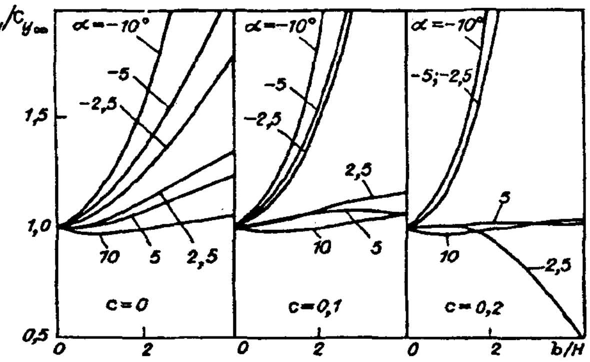


Fig. 3

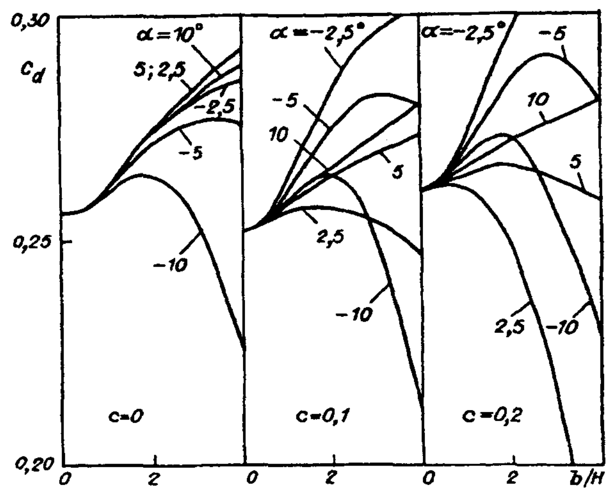


Fig. 4

$$g_k^{(r)}(z_{0k}^{(p)}) = \varkappa e^{-i\vartheta(z_{0k}^{(r)})}\,\mathrm{Im}\left\{\frac{(-1)^{r-1}}{\Delta_k^{(r)}}\ln\frac{z_{0k}^{(p)}-\zeta_{k-1}^{(r)}}{z_{0k}^{(p)}-\zeta_k^{(r)}} - \frac{e^{i\vartheta(z_{0k}^{(r)})}}{z_{0k}^{(p)}-z_k^{(r)}}\right\}. \qquad (1.10)$$

Here $\zeta_{k-1}^{(r)}$, $\zeta_k^{(r)}$ are complex coordinates of the ends of the k-th element of the vortex layer on contour $L_r$; $\Delta_k^{(r)}$ is the length of this element; $\varkappa = 1$ if $|z_{0k}^{(1)} - z_{0k}^{(2)}| + |z_k^{(1)} - z_k^{(2)}| < 2\Delta_k^{(r)}$; $\varkappa = 0$ in other cases.

The function $g_k^{(r)}$ makes it possible to determine the tangential velocity component of the fluid (to the element $[\zeta_{k-1}^{(r)}, \zeta_k^{(r)}] \in L_r$) at the point $z_{0k}^{(p)} \in L_p$ for $p \neq r$ ($p, r = 1, 2$). We note that in formula (1.10), the points $z_{0k}^{(p)}$ should not coincide with the ends of the elements of the vortex layer along $L_p(z_{0k}^{(p)} \neq \zeta_k^{(p)}, p = 1, 2)$.

Substituting expressions (1.7)-(1.10) into (1.5), (1.6) when $z_r = z_{0k}^{(r)}$ ($k = 1, \ldots, N$; $r = 1, 2$) and replacing $V_s(z_{0k}^{(1)}) - V_s(z_{0k}^{(2)})$ by $\Gamma_k^{(2)}/\Delta_k^{(2)} - \Gamma_k^{(1)}/\Delta_k^{(1)}$, we obtain a system of 2N linear algebraic equations in intensities of discrete vortices.

To calculate the distributed and overall aerodynamic characteristics, we developed an algorithm permitting a highly accurate determination of these characteristics for airfoils of any thickness, including a thickness as small as desired. This algorithm makes use of a special approximation of the contour in the immediate vicinity of the leading edge of the airfoil, and the corresponding approximation for the functions $\gamma_1(z_1)$, $\gamma_2(z_2)$ in terms of the given values $\Gamma_1^{(1)}, \ldots, \Gamma_N^{(2)}$.

**2.** The calculation was carried out for a symmetric Joukowski airfoil. The upper and lower sides of the airfoil were divided into N elements of equal length $\Delta$. Discrete vortices were placed at a distance of $\Delta/4$, and the control points, at a distance of $3\Delta/4$ from the origin of each element. On the first elements ($k = 1$), the control point was shifted back by $0.05\Delta$, making it possible to obtain a highly accurate calculation of the intensity of discrete vortices $\Gamma_1^{(1)}$, $\Gamma_1^{(2)}$ in the limiting case of an infinitely thin airfoil [6]. The calculation algorithm was tested with known solutions of the problems of flow of an unlimited stream around a Joukowski airfoil and motion of a plate near a flat screen [2, 7].

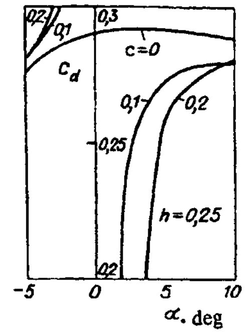


Fig. 5

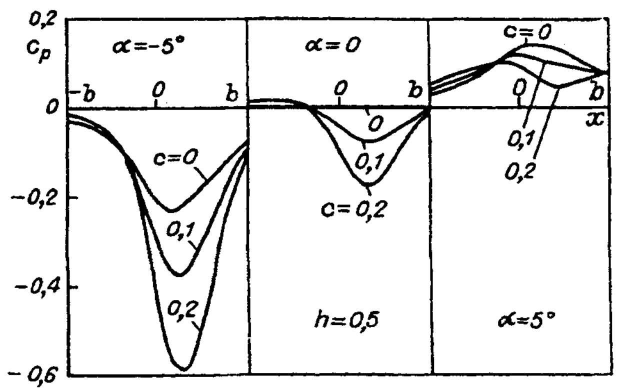


Fig. 6

The numerical experiment was carried out with airfoils of relative thickness c = 0, 0.1, 0.2 over a wide range of angles of attack $\alpha$ and h = H/b. The number of discrete vortices on each side of the airfoil was chosen as N = 40, which in testing the relative calculation error gave < 1%. We calculated the standard aerodynamic coefficients $C_x$, $C_y$, and $C_m$, which determine the overall aerodynamic forces $R_x$, $R_y$ and the moment M about the leading edge of the airfoil, the dimensionless distance of the pressure center from the leading edge $C_d = C_m/(C_y \cos \alpha - C_x \sin \alpha)$, and the distribution of pressure along the screen and airfoil contour – the coefficient $C_p = 2(p - p_\infty)/(\rho V_\infty^2)$, where $\rho$ is the density of the fluid and p and $p_\infty$ are, respectively, the hydrodynamic pressure at the point under consideration and at an infinitely distant point.

The main objective of the numerical experiment was to determine the influence of the relative thickness of the airfoil on its aerodynamic characteristics. Of great interest for practical applications are the dependences of the lift coefficient $C_y$ on the angle of attack and on the distance of the airfoil from the screen. The calculation results presented in Figs. 2 and 3 show a substantial influence of airfoil thickness on these dependences, especially at small angles of attack and a short distance h from the screen. Note that the influence of airfoil thickness on $C_y = C_y(\alpha, h)$ was found to be stronger than in the case of the calculated data in [2], which were obtained in the approximation of linear theory. An analogous nature of the influence of airfoil thickness is observed for the moment coefficient $C_m$ ($C_x = 0$).

Figure 4 shows the position of the pressure center on the airfoil as a function of the parameter b/H at different angles of attack and different relative thicknesses of the airfoil. This dependence at c = 0 (plate) is consistent with the results of [7]. For a solid airfoil (c ≠ 0), the pressure center can move substantially with a small change of the angle of attack near the values of $\alpha$ at which $C_y = 0$ and $C_m = 0$ (Fig. 5).

Wall thickness also strongly affects the distribution of pressure over the lower side of the airfoil and over the screen, especially at negative angles of attack. As an example, Fig. 6 shows the distribution of pressure, generated by an airfoil of relative thickness c = 0, 0.1, 0.2, along the screen. The trailing edge of the airfoil is separated from the screen by a distance equal to one-half the chord (h = 0.5), and the angle of attack is $\alpha$ = 5, 0, 5°. The value x = 0 corresponds to the leading edge of the airfoil, and x = b corresponds to the trailing edge. The above results are in qualitative agreement with the data of [8].

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